Homework 7

Problem 1: The H-R Diagram as a distance indicator

1a) Through stellar parallax, we can obtain the distance to nearby star clusters (like M015 or the Hyades). We create HR disgrams for M105 and a more distant star cluster (like Paschapuppy) by measuring the apparent magnitude and color (which is a proxy for temperature) of a large number of stars in each cluster. For stars on the main sequence, we assume that stars of the same color have the same intrinsic luminosity. Thus, by comparing the difference in apparent magnitude for these stars, we can calculate the distance to the more distant cluster. Since we will have the difference in apparent magnitudes, we will need the equation for apparent magnitudes:

$$m_1 - m2 = -2.5log(\frac{I_1}{I_2}) \tag{1}$$

This will give us the ratio of the intensities for stars in the two clusters. Using the equation for intensity, we can then find the ratio of their distances and solve for the distance to the more distant cluster:

$$I = \frac{L}{4\pi d^2} \tag{2}$$

1b) Mathematically, this problem is very similar to problem 4 from homework 6. From the graph, we see that the main sequence of Paschapuppy is roughly 5 magnitudes fainter for stars of all temperatures. We insert this $\Delta m = m_1 - m_2 = 5$ into the apparent magnitude equation:

$$5 = -2.5log(\frac{I_P}{I_{M105}}) \tag{3}$$

Then we divide both sides by -2.5:

$$-2 = log(\frac{I_P}{I_{M105}}) \tag{4}$$

Removing the logarithm by making both sides a power of 10 leaves us with:

$$10^{-2} = \frac{I_P}{I_{M105}} \tag{5}$$

Next, we use the equation for intensity and plug this into the equation above:

$$10^{-2} = \frac{\frac{L_{MP}}{4\pi d_P^2}}{\frac{L_{M105}}{4\pi d_{M105}^2}} \tag{6}$$

The 4π's in the previous equation cancel. Additionally, we are assuming that stars of the same temperature have the same luminosity. That means the luminosities cancel as well, leaving

$$10^{-2} = \frac{d_{M105}^2}{d_P^2} \tag{7}$$

Isolating d_P on one side of the equation, we get:

$$d_P^2 = 10^2 d_{M105}^2 \tag{8}$$

We know that the distance to the LMC is 1000pc, so we can plug that number in above and solve to get $d_{M137} = 10000pc$ (or 10kpc).

Problem 3: Doppler Shift

The velocity v of a galaxy is determined by the wavelength Doppler shift of light, via redshift z. Reshift is determined by the equation,

$$z = \frac{\lambda_{obs} - \lambda_{lab}}{\lambda_{lab}}$$

In this problem, $\lambda_{obs} = 5500\text{Å}$ and $\lambda_{lab} = 5000\text{Å}$. So our redshift is,

$$z = \frac{5500\text{Å} - 5000\text{Å}}{5000\text{Å}} = \frac{500\text{Å}}{5000\text{Å}} = 0.1$$

To find the velocity this reshift represents, we multiply by the speed of light c, ($v = cz$) which is $3x10^5$ km/s. Thus, Anibal is receding from us at a velocity $v = 3x10^4$ km/s. (We note that in this example, $v < c$. If $v > c$, then we could not use the formula $v = cz$ to determine the velocity.)

To find the distance to Anibal, we use the Hubble law, $v = Hd$. Since v has units of km/s, and H has units of km/s/Mpc, d will have units of Mpc. (Mpc is the abbreviation for megaparsec.) Therefore,

$$d = \frac{v}{H} = \frac{3x10^4\text{km/s}}{50\text{km/s/Mpc}} = 600\text{Mpc}$$

So Anibal is 600 Mpc away from us.

Problem 3: Doppler Shift

(a) Doppler shift is the change in the wavelength (or frequency) of radiation due to the relative motion of the observer and/or the source[1] along the line of sight. A Doppler shift in the spectrum of an astronomical object is commonly known as a redshift when the shift is towards longer wavelengths (the object is moving away) and as a blueshift when the shift is towards shorter wavelengths (the object is approaching). The relativistic Doppler shift is given by the following formula:

$$\frac{\Delta\lambda}{\lambda_o} = \frac{\lambda - \lambda_o}{\lambda_o} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}} - 1 \tag{9}$$

However, when the relative velocity between the source and the observer is very small compared to the velocity of light the preceding formula can be cast into the following simple one:

$$\frac{\Delta\lambda}{\lambda_o} = \frac{\lambda - \lambda_o}{\lambda_o} = \frac{v}{c} \tag{10}$$

where λ and λ_o are the observed and rest wavelength respectfully, v is the relative velocity between the observer and the source and c is the speed of light[2].

(b) The rest wavelength the radar gun uses is: $\lambda_o = 10\ m$. The observed (shifted) wavelength is: $\lambda = 10.000001\ m$. That's all the information we need to find the velocity of Justin's truck:

1. The change in wavelength is: $\lambda - \lambda_o = 10.000001\ m\ -\ 10\ m = 0.000001\ m$. Since it is positive, it corresponds to a redshift and hence Justin is driving away from Sheriff Catchem.
2. Now a direct application of eq. 11 yields:

$$\begin{aligned} \frac{10.000001\ m\ -\ 10\ m}{10\ m} &= \frac{v}{3\cdot 10^8 m/s} => \\ v &= \frac{0.000001\ m}{10\ m} 3\cdot 10^8\ m/s => \\ v &= \frac{10^{-6}}{10} 3\cdot 10^8\ m/s => \end{aligned}$$

[1] For the purpose of this class the velocity of the observer is always zero and hence Doppler shifts are produced due to the motion of the source.

[2] $c = 3\cdot 10^8 m/s = 3\cdot 10^5 km/s = 3\cdot 10^{10} cm/s$

$$\begin{aligned} v &= 3 \cdot 10^8 \cdot 10^{-7}\ m/s => \\ v &= 30\ m/s \end{aligned} \tag{11}$$